

Direct Estimation of Spinal Cobb Angles by Structured Multi-output Regression

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Abstract. The Cobb angle that quantitatively evaluates the spinal curvature plays an important role in the scoliosis diagnosis and treatment. Conventional measurement of these angles suffers from huge variability and low reliability due to intensive manual intervention. However, since there exist high ambiguity and variability around boundaries of vertebrae, it is challenging to obtain Cobb angles automatically. In this paper, we formulate the estimation of the Cobb angles from spinal X-rays as a multi-output regression task. We propose structured support vector regression (S²VR) to jointly estimate Cobb angles and landmarks of the spine in X-rays in one single framework. The proposed S²VR can faithfully handle the nonlinear relationship between input images and quantitative outputs, while explicitly capturing the intrinsic correlation of outputs. We introduce the manifold regularization to exploit the geometry of the output space. We propose learning the kernel in S²VR by kernel alignment to enhance its discriminative ability. The proposed method is evaluated on the spinal X-rays dataset of 439 scoliosis subjects, which achieves the inspiring correlation coefficient of 92.76% with ground truth obtained manually by human experts and outperforms two baseline methods. Our method achieves the direct estimation of Cobb angles with high accuracy, indicating its great potential in clinical use.

1 Introduction

Cobb angles are widely used for scoliosis diagnosis and treatment decisions in clinical routine. Scoliosis is a structural, lateral, rotated curvature of the spine, which especially arises in children at or around puberty and leads to disability [1]. For clinical examination of scoliosis, the radiography (X-ray) is the most common imaging technique with the cheap acquisition and less time cost [2]. Cobb angles (Fig. 1(a)) derived from a posteroanterior (back to front) X-ray and manually measured by selecting the most tilted vertebra at the top and bottom of the spine respect to the horizontal line are faithfully used to quantify the magnitude of spinal deformities [3].

However, conventional manual measurement involves the heavy intervention of identifying the vertebrae and measuring angles, it suffers from huge variability

and high unreliability while being labor-intensive. The accuracy of measurement is affected by many factors, such as the selection of vertebrae, the bias of different observers and inaccurate protractors.

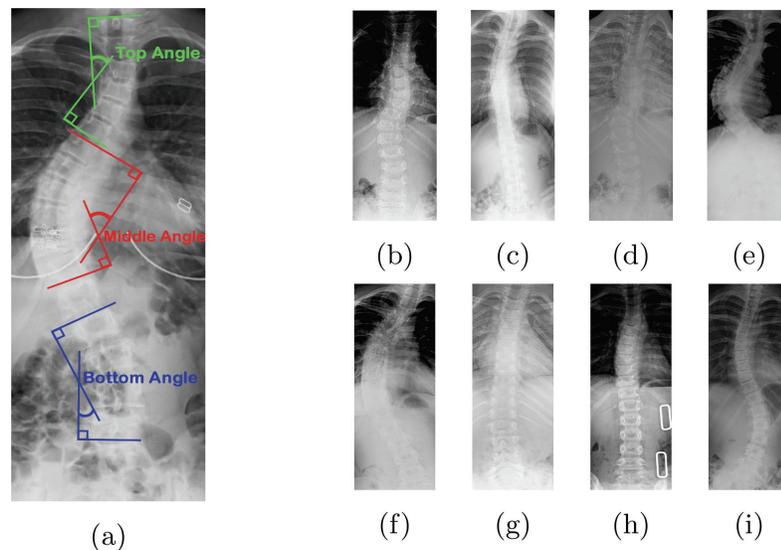


Fig. 1. It is challenging to measure three Cobb angles in (a) due to high ambiguity and variability in scoliosis X-rays from different subjects (a)–(i).

It is challenging to estimate Cobb angles automatically due to the high ambiguity and variability of X-rays. As shown in Fig. 1, large anatomical variability and low tissue contrast can lead to the complexity of identification of interesting vertebrae and further measurement. Active contour model [4], customized filter [5] and charged-particle model [6] were used for localizing vertebrae. Angles are computed from the slope of these vertebrae. Although these methods can derive the Cobb angles automatically, they suffer from the complexity of multiple processing stages, which is computationally expensive. Moreover, segmentation for multiple objects of all vertebrae is extremely challenging, especially in spinal X-rays with variant structures and ambiguous boundaries.

Without filtering or segmentation, direct estimation methods [7–11] based on statistical learning, which achieves the great success of large-scale image data recently, have the potential for Cobb angles estimation. Compared with conventional segmentation methods, such as variational models [12], direct estimation methods characterize the relationship between the appearance of images and high-level semantic concepts via supervised learning, which enables efficient computation and compact structure modeling. By exploring data statistics, direct methods are robust and can handle fuzzy boundaries and region heterogeneity.

The estimation of the Cobb angles can benefit from the prediction of coordinates of landmarks. Due to the ambiguity and variability of spinal images, it is challenging to obtain the angles separately. Intuitively, the Cobb angles closely

correspond to the spinal shape which can be substantially characterized by landmarks. In this paper, we train the model by minimizing the loss with respect to both the Cobb angles and all coordinates of landmarks simultaneously, which demonstrates the robustness of our method. In contrast to existing direct methods [7–10], the proposed method jointly estimates landmarks and Cobb angles, which not only improves the prediction accuracy but also provides an intuitive validation of angle estimation for the radiologist.

Although existing direct methods achieved great success, their direct application to our task suffers from three major limitations. First, these methods fail to explicitly characterize the correlations among outputs, which is essential for discriminative learning. Second, they neglect the intrinsic geometry of the distribution of output values, which would result in the non-smooth solution and then has a negative effect on the angle estimation. Third, the kernel parameters, e.g., the bandwidth, are usually set manually, which does not guarantee to be optimal for different applications.

In this paper, we propose structured support vector regression (S^2VR) to directly predict the Cobb angles and landmarks of a spine from its X-ray. In comparison to conventional support vector regression (SVR), the S^2VR accomplishes of nonlinear mapping and explicit correlation modeling in one single framework. In particular, the nonlinear mapping handles the relationship between input images and high-level outputs (angles and landmarks), while the explicit correlation modeling captures the correlations among outputs.

In summary, our work contributes to the following aspects:

- We achieve the jointly automatic estimation of Cobb angles and spinal landmarks by multi-output regression, which is essential for the evaluation of spinal curvature from radiography directly.
- We propose a novel multi-output regression model called S^2VR , which can simultaneously handle the nonlinear input-output relationship and intrinsic inter-output correlation. The framework is endowed with great generality which enables wide applications for other measurement estimation.
- We introduce a manifold regularizer into the framework by exploring the geometry structure of the output space, which enables smooth solutions.
- To improve the performance, we propose to learn the kernels by using kernel target alignment, which increase the discriminative ability of the kernels.

2 Methodology

The proposed structured support vector regression (S^2VR) directly estimates the Cobb angles and coordinates of landmarks by formulating the prediction as a multi-output regression task. S^2VR consists of nonlinear mapping process and explicit correlation learning stage. The manifold regularization is introduced to explore the intrinsic geometry of the output space. Since our method is built on support vector regression, we learn the kernel based on kernel target alignment.

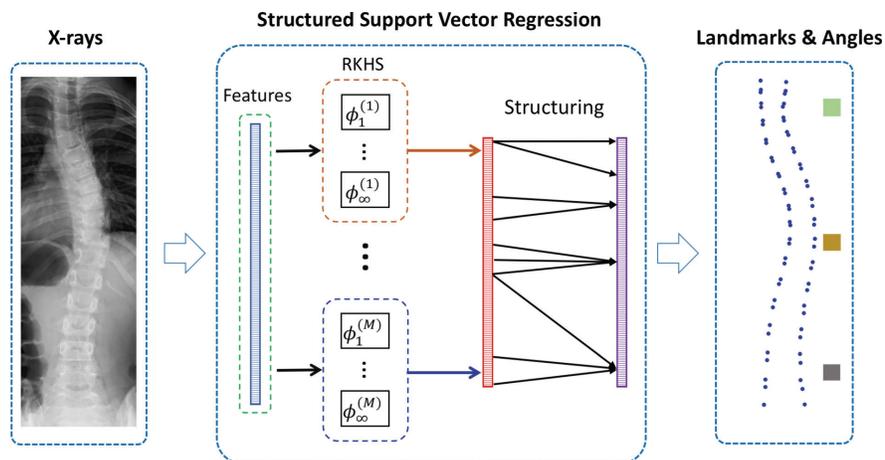


Fig. 2. The framework of the structured support vector regression (S^2VR).

2.1 Structured Support Vector Regression

Within the proposed S^2VR , we extract image features of a X-ray that denoted as $\mathbf{x}_i \in \mathbb{R}^d$. The coordinates and angles are represented by $\mathbf{y}_i \in \mathbb{R}^q$ and $\mathbf{y}_i = [h_1, \dots, h_c, v_1, \dots, v_c, a_1, a_2, a_3]$, c is the number of landmarks, h_i and v_i are the horizontal and vertical axes of i -th landmark point. In addition, the number of Cobb angles is 3, thus, $q = 2c + 3$. The regression task is to predict the coordinates and the Cobb angles from the input features. The framework of S^2VR is illustrated in Fig. 2.

SVR for Nonlinear Input-Output Relationship Learning. Support vector regression (SVR) has been widely used to handle complex nonlinear relationships. Its effectiveness is guaranteed by the sparseness of solutions. The SVR model takes the following optimization formulation:

$$\arg \min L(W, \mathbf{b}) = \frac{1}{2} \|W\|^2 + \tau \sum_{i=1}^N \nu(u_i) \quad (1)$$

where

$$\nu(u_i) = \begin{cases} 0 & u_i < \varepsilon \\ (u_i - \varepsilon)^2 & u_i \geq \varepsilon \end{cases}, \quad (2)$$

$u_i = \|\mathbf{y}_i - f_p(\mathbf{x}_i)\|^2$, $f_p(\mathbf{x}_i) = W\phi(\mathbf{x}_i) + \mathbf{b}$, $\|\cdot\|$ is ℓ^2 -norm, $\phi: \mathbb{R}^d \rightarrow \mathbb{R}^H$ is a nonlinear transformation to a higher H -dimensional space, $W \in \mathbb{R}^{q \times H}$, $\mathbf{b} \in \mathbb{R}^q$, $\phi(\mathbf{x}_i) \in \mathbb{R}^H$, $\mathbf{y}_i \in \mathbb{R}^q$, τ and N denote a positive constant of the tuning parameter and the number of training samples, respectively.

S^2VR for Explicit Inter-output Correlation Learning. We introduce a structure matrix S , $S \in \mathbb{R}^{q \times q}$ to capture the intrinsic correlations of outputs

(angles and landmarks). The prediction model becomes $f(\mathbf{x}_i) = S(W\phi(\mathbf{x}_i) + \mathbf{b})$ and the S^2VR model can be defined as:

$$L(W, \mathbf{b}, S) = \frac{1}{2}\|W\|^2 + \tau \sum_{i=1}^N \nu(u_i) + \lambda \|S^\top\|_{2,1}, \quad (3)$$

where $u_i = \|\mathbf{y}_i - f(\mathbf{x}_i)\|$, $\|S\|_{2,1} = \sum_{i=1}^q \sqrt{\sum_{j=1}^q S_{ij}^2}$ and λ is the tuning parameter.

Advantages: The appealing property of structure matrix S via $\ell_{2,1}$ -norm regularization is to encourage multiple outputs to share the similar sparsity patterns of parameters. Therefore, the structure, such as spatial correlations which share similar specific patterns, can be captured for the robust prediction of angles and coordinates.

Manifold Regularization for S^2VR . To further improve the performance of the proposed algorithm, we consider incorporating a manifold regularization [13] term based on a graph Laplacian to model the local geometrical structure of the output space. It has been shown that learning performance can be significantly enhanced if the geometrical structure is exploited and the local invariance is considered.

We adopt the loss function with the additional regularization as:

$$L(W, \mathbf{b}, S) = \frac{1}{2}\|W\|^2 + \frac{\gamma}{2} \text{Tr}(FGF^\top) + \tau \sum_{i=1}^n \nu(u_i) + \lambda \|S^\top\|_{2,1}, \quad (4)$$

where $G \in \mathbb{R}^{N \times N}$ is the graph Laplacian matrix [14] and $F = [f(\mathbf{x}_1), \dots, f(\mathbf{x}_N)]^\top$.

Here we select the unnormalized graph Laplacian matrix given by $G = \hat{D} - E$ to represent the labels $\mathbf{y}_i|_{i=1:N}$ of data in form of the similarity graph. We simply connect all points with positive similarity with each other, and we weight all edges by E_{ij} . The similarity function is the Gaussian function $E_{ij} = \exp(-\|\mathbf{y}_i - \mathbf{y}_j\|^2 / (2\rho^2))$, where the parameter ρ is the width of the function. \hat{D} is the diagonal matrix given by $\hat{D}_{ii} = \sum_{j=1}^N E_{ij}$.

Advantages: The manifold regularization exploits the geometry of the distribution of output data. In particular, due to the high ambiguity and variability of image appearance, distributions of input features and output angles (landmarks) could be inconsistent in estimation. Therefore, by leveraging the strength of manifold regularization, we ensure the smoothness of solution with respect to the output space.

2.2 Kernelization

Since the relationship between inputs of image features and outputs of angles (landmarks) is complex and highly nonlinear, we kernelize the (4) to conduct nonlinear regression.

Kernel S²VR. Before kernelization, we rewrite (4) in term of traces:

$$L(W, \mathbf{b}, S) = \frac{1}{2} \text{Tr}(W^\top W) + \frac{\gamma}{2} \text{Tr}(FGF^\top) + \tau \text{Tr}(UE^\top E) + \lambda \|S^\top\|_{2,1}, \quad (5)$$

where $E = [\mathbf{e}_1, \dots, \mathbf{e}_N]$, $\mathbf{e}_i = \mathbf{y}_i - S(W\phi(\mathbf{x}_i) + \mathbf{b})$, and U is the diagonal matrix given by

$$U_{ii} = \begin{cases} 0 & u_i < \varepsilon \\ 1 & u_i \geq \varepsilon \end{cases}. \quad (6)$$

Assume that the mapping function $\phi(\mathbf{x}_i)$ is in some RKHS of infinite dimensionality. According to the Representer Theorem [15], $W = \beta \Phi^\top(X)$, where $\beta \in \mathbb{R}^{q \times N}$ and $X = [\mathbf{x}_1, \dots, \mathbf{x}_N]$. The kernel matrix is defined as $K = \Phi^\top(X)\Phi(X)$. We rewrite (5) with the kernel version:

$$Q(\beta, S) = \frac{1}{2} \text{Tr}(\beta K \beta^\top) + \frac{\gamma}{2} \text{Tr}(S \beta K G K \beta^\top S^\top) + \frac{1}{2} \text{Tr}(UE^\top E) + \lambda \|S^\top\|_{2,1}, \quad (7)$$

where

$$E = Y - S \beta K. \quad (8)$$

Discriminant Kernel Learning. We focus on learning a cone combination of given base kernels $K_z|_{z=1:Z}$ to obtain an efficient kernel. Therefore, for the summation formulation of $K = \sum_{z=1}^Z \mu_z K_z$, we propose learning the weight coefficients $\boldsymbol{\mu}$ associated with basic kernels by kernel alignment, which has shown great effectiveness in learning the optimal combination of multiple kernels [16, 17].

Kernel alignment is proposed to align an input kernel K to a target kernel K_T by maximizing the similarity between them, which offers a best-suited way to obtain the weight coefficients $\boldsymbol{\mu}$. We now introduce the kernel alignment formulation to learn the kernel in S²VR. We would like to maximize the alignment $A(K_\mu, K_T)$ between the target kernel matrix K_T and the kernel K_μ . The optimization problem can be formulated as

$$\boldsymbol{\mu}^* = \arg \max A(K_\mu, K_T) = \arg \max \frac{\text{Tr}(K_\mu K_T)}{\sqrt{\text{Tr}(K_T K_T) \text{Tr}(K_\mu K_\mu)}}. \quad (9)$$

The target kernel matrix K_T is constructed by defining the target kernel $K_T = Y^\top Y$, where $Y = [\mathbf{y}_1, \dots, \mathbf{y}_N]$.

The objective function in (9) can be equivalently rewritten as follows:

$$\boldsymbol{\mu}^* = \arg \max_{\|\boldsymbol{\mu}\|=1, \boldsymbol{\mu} \geq 0} \frac{\boldsymbol{\mu}^\top \boldsymbol{\alpha} \boldsymbol{\alpha}^\top \boldsymbol{\mu}}{\boldsymbol{\mu}^\top V \boldsymbol{\mu}} \quad (10)$$

where $\|\boldsymbol{\mu}\| = 1$ is a regularization term, $\boldsymbol{\mu} \geq 0$ guarantees the positive definiteness, for $i, j \in \{1, \dots, Z\}$, $\boldsymbol{\alpha}$ is defined by $\alpha_i = \text{Tr}(K_i K_T)$ and the matrix V is defined by $V_{ij} = \text{Tr}(K_i K_j)$. By following [18], we resort to a more efficient quadratic programming problem to solve (10).

Advantages: The kernels in S²VR are learned in a supervised way via kernel alignment, which increases the discriminative ability of the kernels. Also, it is efficient that parameters in kernel functions are learned automatically rather than tuned by cross validation.

2.3 Alternating Optimization

S²VR is efficiently optimized by using a new alternating optimization method. The proposed method is fast, whose convergence is theoretically analyzed. Minimization (7) can be decomposed into two sub-problems with respect to the kernel coefficients β and the structure matrix S .

Fixing S , the minimization problem with respect to β can be optimized by iteratively reweighted least squares (IRWLS) [19]. We construct a quadratic approximation of (5) about u_i :

$$Q(W, \mathbf{b}) = \frac{1}{2} \text{Tr}(W^\top W) + \frac{\gamma}{2} \text{Tr}(FGF^\top) + \frac{1}{2} \text{Tr}(DE^\top E) + \tau T, \quad (11)$$

where D is the diagonal matrix given by

$$D_{ii} = \frac{\tau}{u_i} \left. \frac{dL(u)}{du} \right|_{u_i} = \begin{cases} 0 & u_i < \varepsilon \\ \frac{2\tau(u_i - \varepsilon)}{u_i} & u_i \geq \varepsilon \end{cases} \quad (12)$$

and τT is a sum of constant terms that do not depend either on W or \mathbf{b} . We modify (11) in the kernel version:

$$Q(\beta) = \frac{1}{2} \text{Tr}(\beta K \beta^\top) + \frac{\gamma}{2} \text{Tr}(S \beta K L K \beta^\top S^\top) + \frac{1}{2} \text{Tr}(DE^\top E) + \tau T, \quad (13)$$

where $E = Y - S\beta K$. To obtain β , we can equate its gradient to zero:

$$\frac{\partial Q}{\partial \beta} = \beta K - \gamma S^\top S \beta K G K - S^\top (Y - S\beta K) D K = 0, \quad (14)$$

$$(S^\top S)^{-1} \beta + \beta K (\gamma G + D) = S^{-1} Y D. \quad (15)$$

It is the standard Sylvester equation ($A\beta + \beta B = C$, where $A = (S^\top S)^{-1}$, $B = K(\gamma G + D)$ and $C = S^{-1} Y D$).

We minimize (13) by constructing a descending direction using the optimal solution of (15), and the next step solution is computed by a line search algorithm [18]. Inspired by the IRWLS, we propose the adapted IRWLS procedure summarized in Algorithm 1.

Given fixed β , the derivative of (7) respect to S is computed as:

$$\frac{\partial Q}{\partial S} = 2\lambda S P - \tau (Y - S\beta K) D (\beta K)^\top + \gamma S \beta K G K \beta^\top = 0 \quad (16)$$

$$S = \tau Y D K \beta^\top \left(2\lambda P + \tau \beta K D K \beta^\top + \gamma \beta K G K \beta^\top \right)^{-1}, \quad (17)$$

Algorithm 1. Adapted IRWLS

-
- 1: Initialization: set $k = 0$, compute D^k using (12)
 - 2: **repeat**
 - 3: Compute β^k in (15) by using the Sylvester equation, and denote it as β^ℓ . Get the descending direction as $\Delta^k = \beta^\ell - \beta^k$.
 - 4: Update $\beta^{k+1} = \beta^k + \eta^k \Delta^k$, and the step size η^k is computed by using the backtracking algorithm.
 - 5: Obtain D^{k+1} .
 - 6: Set $k = k + 1$.
 - 7: **until** Convergence
-

Algorithm 2. Iteratively Computing S

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- 1: Initialization: set $k = 0$
 - 2: **repeat**
 - 3: Compute the diagonal matrix P^k .
 - 4: Update S^{k+1} using (17).
 - 5: Set $k = k + 1$.
 - 6: **until** Convergence
-

where S is a diagonal matrix with $P_{jj} = \frac{1}{2\|S_j\|_2}$, $P \in \mathbb{R}^{q \times q}$ and S_j is the column vector of S .

We compute the derivative of S using (17). The calculation of P is based on S . Therefore, the new S is computed by using the current S . The procedure is summarized in Algorithm 2. Therefore, minimization (7) can be solved by alternatively optimizing β and S .

Proof of Convergence: The fast convergence for optimization ensures the efficiency of our method. We provide the theoretical analysis with rigorous proof for the convergence of the optimization algorithm. Since the optimization is conducted in an alternating iterative way, we first prove the convergence for the adapted IRWLS procedure for optimizing β .

Theorem 1. $Q(\beta)$ in (13) bounds together all outputs and monotonically decreases in each iteration of Algorithm 1

Proof. The proof can be referred to the proof of Appendix in [19].

Now that the convergence for the adapted IRWLS procedure is guaranteed in Theorem 1, the following theorem shows the convergence of the alternating iterative method.

Theorem 2. $Q(\beta, S)$ in (7) is bounded from below and monotonically decreases with each alternating iterative optimization step for β and S .

Proof. Due to the sum formation of $Q(\beta, S)$ in (13), we have the bound of $Q(\beta, S) \geq 0$. Let $\beta^{(t)}$ and $S^{(t)}$ denote β and S in the t -th iteration respectively. Then, $\beta^{(t)}$ and $S^{(t)}$ are computed by $\beta^{(t)} \leftarrow \arg \min_{\beta} Q(\beta, S^{(t-1)})$ and

$S^{(t)} \leftarrow \arg \min_S Q(\beta^{(t-1)}, S)$. Since $Q(\beta, S^{(t-1)}) \geq Q(\beta, S^{(t)})$ has been proved in [20], we obtain the following inequality:

$$\dots \geq Q(\beta^{(t-1)}, S^{(t-1)}) \geq Q(\beta^{(t)}, S^{(t-1)}) \geq Q(\beta^{(t)}, S^{(t)}) \geq \dots \quad (18)$$

$Q(\beta^{(t)}, S^{(t)})$ is monotonically decreasing as $t \rightarrow +\infty$. Therefore, the convergence of the optimization of $Q(\beta, S)$ is proved.

2.4 Prediction

In the testing stage, given a new input $\mathbf{x}^{(t)}$, the predicted angles and landmarks are given by $\hat{\mathbf{y}}^{(t)} = S\beta K^{(t)}$, where $K^{(t)} = \sum_{z=1}^Z \mu_z K_z^{(t)}$, $K_z^{(t)} = \Phi^\top(\hat{X})\phi(\mathbf{x}^{(t)})$ and \hat{X} is the matrix composed of the support vectors.

3 Experiments

S²VR has been validated on the spinal X-ray dataset with a large number of subjects. Extensive experiments show that our method with significant effectiveness consistently outperforms the baseline methods, which can be practically used in clinical scoliosis analysis.

3.1 Datasets and Implementation Details

The posteroanterior spinal X-ray dataset is composed of 439 samples from different individuals. Since the cervical vertebrae (the vertebrae of the neck) are seldom involved in spinal deformity [21], we select 17 vertebrae composed of the thoracic spine and lumbar spine for spinal shape characterization. And each vertebra is located by four landmarks with respect to four corners. The landmarks of a spine consist of $c = 68$ points which are annotated manually, while the Cobb angles with 3 values (from top to bottom: TA, MA, BA) are measured by human experts. We choose histogram of oriented gradient (HOG) descriptor [22] as the input of the model due to its strength in local representation. The Gaussian Kernel is selected as the basic kernel and the σ parameter is in the range of $[0.1, 1]$. Our model has been validated using the leave-one-out cross validation scheme.

3.2 Results

The S²VR achieves desirable performance on the spinal X-ray dataset and demonstrates great effectiveness for the Cobb angle estimation and landmark detection. We show the qualitative results of landmark detection in Fig. 3. We also compare with two baseline methods, which shows the strength of our method on the spinal curvature analysis.

Comparison. As demonstrated in Table 1, we compare with two baseline methods, i.e. support vector regression (SVR) [19], shape regression machine (SRM)

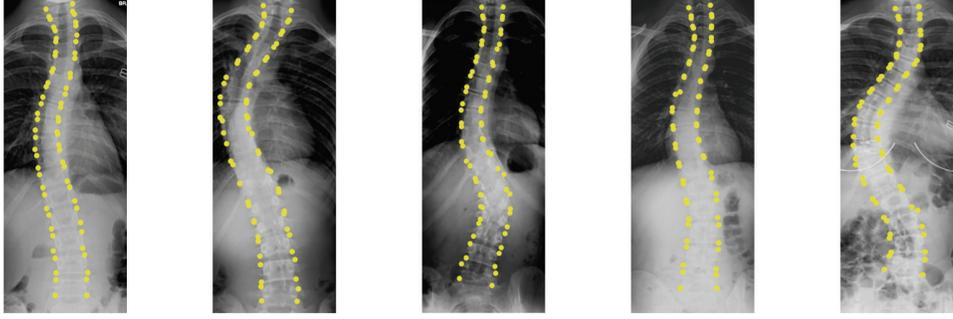


Fig. 3. Our method overcomes huge variations and high ambiguities and achieves high accuracy in landmark detection.

[23] about the relative root mean squared error (RRMSE) and the correlation coefficient, which shows the effectiveness of our method. For vectors of the length of l , let \hat{y} , y , \bar{Y} denotes the ground truth, the estimated vector, the mean of target variable Y over train data, respectively, the RRMSE is defined as

$$R_r = \frac{\sum_{i=1}^l \hat{y}_i - y_i}{\sum_{i=1}^l \bar{Y}_i - y_i}. \quad (19)$$

The S^2VR achieves the lowest average RRMSE of 21.63 and the highest correlation coefficient of 92.76. Since there are strong correlations between outputs, explicit correlation learning in the S^2VR can faithfully exploit the structure information for a better result. Also, the learned and more robust kernel is powerful enough to handle the correspondence between the image appearance and high-level semantic concepts. We also compare the separated prediction with joint prediction. As shown in two tables, the joint prediction consistently outperforms than the separated one.

Table 1. The comparison of the average RRMSE (%) and the correlation coefficient (%) of three angles.

Method	Average RRMSE (%)		Correlation Coefficient (%)	
	Angles	Angles & Landmarks	Angles	Angles & Landmarks
SVR [19]	25.01	23.72	90.83	91.95
SRM [23]	24.83	23.35	91.30	92.13
S^2VR (Ours)	23.69	21.63	91.94	92.76

Effectiveness. The effectiveness of the proposed method is demonstrated by the outstanding performance on the spinal X-ray dataset. The estimated Cobb angles by the proposed method are compared with those by manual measurement. The correlations between estimated angles and ground truth are depicted in Fig. 4, respectively. The proposed method achieves a correlation coefficient of 0.923 for the middle angle and can yield 0.884 and 0.902 for the rest two angles.

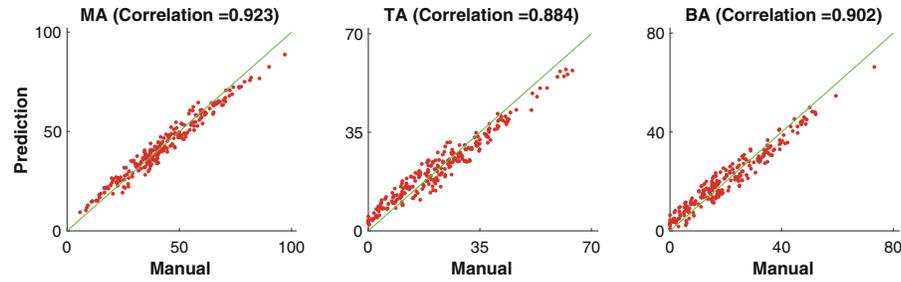


Fig. 4. The correlation coefficients between three angles predicted by the proposed method and ground truth for the MA, TA and BA, respectively. The method achieves a high correlation coefficient with manual measurement.

4 Conclusion

In this paper, we propose structured support vector regression (S^2VR) to directly predict the Cobb angles and landmarks in spinal X-rays from image features. The proposed S^2VR consists of non-linear mapping and explicit structure modeling, which can handle the highly nonlinear relationship between features and quantitative evaluation parameters and explicitly learn the inter-output corrections. Moreover, the manifold regularization is introduced for the smooth solution. To obtain a discriminative kernel, we propose to learn the kernel in S^2VR by kernel target alignment, which can leverage the strength of supervised kernel learning. Extensive experiments on the spinal X-ray dataset show the great effectiveness of our method compared with two baseline methods.

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